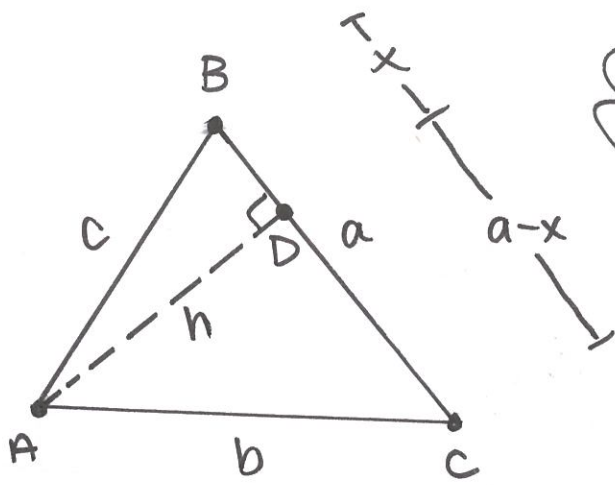




Proof of Law of cosines

Given any $\triangle ABC$,
construct an altitude

from any vertex. Let h
be the length of the altitude.
This breaks up $\triangle ABC$ into smaller
right $\triangle$ s. Let D be the point
where the altitude intersects the side
of the $\triangle$. Let x be the length
of BD , so $DC = a - x$.

applying the Pyth. Thm to $\triangle ABD$,
we have

$$h^2 + x^2 = c^2$$

and applying trig. ratios to $\triangle ABD$, we
have

$$\cos B = \frac{x}{c}, \text{ so } x = c(\cos B)$$

applying the Pyth. Thm to $\triangle ADC$, we
have

$$(a-x)^2 + h^2 = b^2$$

$$(a-x)(a-x) + h^2 = b^2$$
$$a^2 - 2ax + x^2 + h^2 = b^2$$

using substitution, we have the following:

$$a^2 - 2a(c)(\cos B) + c^2 = b^2$$

So,

$$b^2 = a^2 + c^2 - 2ac(\cos B)$$

The same approach is applied when the altitude is constructed from the remaining two vertices to obtain the equations

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$